Appendix A: Computing Scale Factors and Alignment Vectors

This appendix shows a solution method for the system of equations derived in the main body of the patent, to obtain the scale factors and the alignment vectors of the sensitive axes of the multi-axis accelerometer device.

In Orientation 1:

$$P_{A,1}= \kappa*\alpha_A*g*\sin(\theta)*\exp(i*\phi_1)*(A_x - i*A_y) \quad (1)$$

$$P_{B,1}= \kappa*\alpha_B*g*\sin(\theta)*\exp(i*\phi_1)*(B_x - i*B_y) \quad (2)$$

$$P_{C,1}= \kappa*\alpha_C*g*\sin(\theta)*\exp(i*\phi_1)*(C_x - i*C_y) \quad (3)$$

In Orientation 2:

$$P_{A,2}= \kappa*\alpha_A*g*\sin(\theta)*\exp(i*\phi_2)*(A_x + i*A_z) \quad (4)$$

$$P_{B,2}= \kappa*\alpha_B*g*\sin(\theta)*\exp(i*\phi_2)*(B_x + i*B_z) \quad (5)$$

$$P_{C,2}= \kappa*\alpha_C*g*\sin(\theta)*\exp(i*\phi_2)*(C_x + i*C_z) \quad (6)$$

In Orientation 3:

$$P_{A,3}= \kappa*\alpha_A*g*\sin(\theta)*\exp(i*\phi_3)*(A_z + i*A_y) \quad (7)$$

$$P_{B,3}= \kappa*\alpha_B*g*\sin(\theta)*\exp(i*\phi_3)*(B_z + i*B_y) \quad (8)$$

$$P_{C,3}= \kappa*\alpha_C*g*\sin(\theta)*\exp(i*\phi_3)*(C_z + i*C_y) \quad (9)$$

Ideal Accelerometer with sensitive axis parallel to the plane of rotation:

$$P\_nominal = \kappa*\alpha\_nominal*g\_nominal*\sin(\theta\_measured) \quad (10)$$

Additionally, the peak DFT values are known from the recorded data and generated ideal data.

In the initial stage, the value of κ can be determined from equation (10), by substituting the known values for P_nominal, α_nominal, g_nominal and θ_measured.

In the next stage, the absolute value of α_A is calculated. This is the scale factor of accelerometer A. By squaring (1), the following equation is obtained:

$$|P_{A,1}|^2 = |\kappa|^2 * (\alpha_A * g * \sin(\theta))^2 * |\exp(i*\phi_1)|^2 * |(A_x - i*A_y)|^2 \quad (11)$$

$$\therefore |P_{A,1}|^2 = |\kappa|^2 * (\alpha_A * g * \sin(\theta))^2 * \{(A_x)^2 + (A_y)^2\} \quad (12)$$

Similarly, by squaring (4):

$$|P_{A,2}|^2 = |\kappa|^2 * (\alpha_A * g * \sin(\theta))^2 * \{(A_x)^2 + (A_z)^2\} \quad (13)$$

Also, by squaring (7):

$$|P_{A,3}|^2 = |\kappa|^2 * (\alpha_A * g * \sin(\theta))^2 * \{(A_z)^2 + (A_y)^2\} \quad (14)$$

Adding (12), (13) and (14) gives

$$|P_{A,1}|^2 + |P_{A,2}|^2 + |P_{A,3}|^2 = |\kappa|^2 * (\alpha_A * g * \sin(\theta))^2 * \{(A_x)^2 + (A_y)^2 + (A_x)^2 + (A_z)^2 + (A_z)^2 + (A_y)^2\} \quad (15)$$

However, since A_x, A_y, and A_z form an alignment vector $[A_x, A_y, A_z]$, which is a unit vector, the following identity holds:

$$[A_x, A_y, A_z].[A_x, A_y, A_z] = 1 \quad (16)$$

This yields the identity

$$(A_x)^2 + (A_y)^2 + (A_z)^2 = 1 \quad (17)$$

Substituting (17) into (15) gives

$$|P_{A,1}|^2 + |P_{A,2}|^2 + |P_{A,3}|^2 = |\kappa|^2 * (\alpha_A * g * \sin(\theta))^2 * \{2\} \quad (18)$$

The value α_A can be obtained by substituting the values for $P_{A,1}$, $P_{A,2}$, $P_{A,3,}$ and θ, as well as the value for κ from solving (10), into equation (18) and solving the equation (18) with the knowledge that α_A is positive.

The scale factors of accelerometer B and accelerometer C, which are α_B and α_C respectively, can be obtained in a similar manner. Thereby, the scale factors of the sensitive axes of the multi-axis accelerometer device are obtained.

In the final stage the alignment vectors are computed.

Dividing (1) by (2) and rearranging gives:

$$P_{A,1}*\alpha_B*(B_x - i*B_y) - P_{B,1}*\alpha_A*(A_x - i*A_y) = 0 \qquad (19)$$

Similarly dividing (2) by (3) and rearranging gives:

$$P_{B,1}*\alpha_C*(C_x - i*C_y) - P_{C,1}*\alpha_B*(B_x - i*B_y) = 0 \qquad (20)$$

Dividing (4) by (5) and rearranging gives:

$$P_{A,2}*\alpha_B*(B_x + i*B_z) - P_{B,2}*\alpha_A*(A_x + i*A_z) = 0 \qquad (21)$$

Similarly dividing (5) by (6) and rearranging gives:

$$P_{B,2}*\alpha_C*(C_x + i*C_z) - P_{C,2}*\alpha_B*(B_x + i*B_z) = 0 \qquad (22)$$

Dividing (7) by (8) and rearranging gives:

$$P_{A,3}*\alpha_B*(B_z + i*B_y) - P_{B,3}*\alpha_A*(A_z + i*A_y) = 0 \qquad (23)$$

Similarly dividing (5) by (6) and rearranging gives:

$$P_{B,3}*\alpha_C*(C_x + i*C_z) - P_{C,3}*\alpha_B*(B_x + i*B_z) = 0 \qquad (24)$$

Additionally, $[A_{x,} A_{y,} A_z]$, $[B_{x,} B_{y,} B_z]$ and $[C_{x,} C_{y,} C_z]$ are unit vectors. Rearranging (17) gives the following equation:

$$(A_x)^2 + (A_y)^2 + (A_z)^2 - 1 = 0 \qquad (25)$$

Similarly, the following equations can be derived:

$$(B_x)^2+(B_y)^2+(B_z)^2-1=0 \tag{26}$$

$$(C_x)^2+(C_y)^2+(C_z)^2-1=0 \tag{27}$$

The set of equations, (19) through (27), can be solved for the alignment vectors, $[A_x, A_y, A_z]$, $[B_x, B_y, B_z]$ and $[C_x, C_y, C_z]$, using a multidimensional Newton-Raphson solution method. The nominal values of the alignment vectors can be used as initial values for the Newton-Raphson method.

Thus, the values of the scale factors and the alignment vectors of the sensitive axes of the multi-axis accelerometer device can be obtained.